Distributed Model Predictive Control for Heterogeneous Vehicle Platoons under Unidirectional Topologies

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18 Nov. 2015

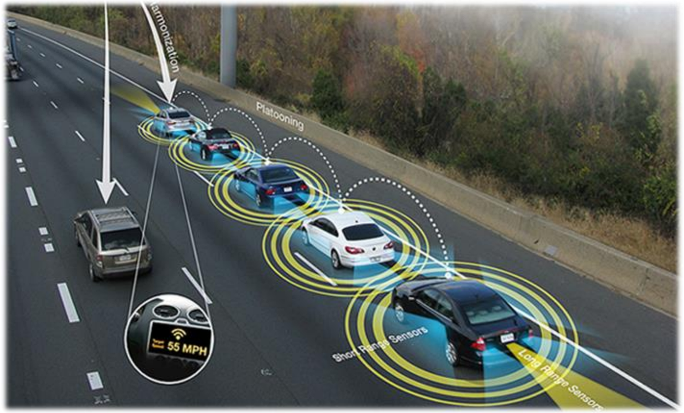
Outline

1. Introduction

Vehicular Platoon

Control Objectives

a) to ensure all the vehicles in the same group to move at the same speed with the leader
b) to maintain the desired spaces between adjacent vehicles



Potential Benefits

- Improve traffic efficiency, enhance road safety, and reduce fuel consumption, etc.
- The earliest implementation can date back to the PATH program during the last eighties

Real-world experiments

USA - PATH

Europe - SARTRE

Japan - Energy ITS

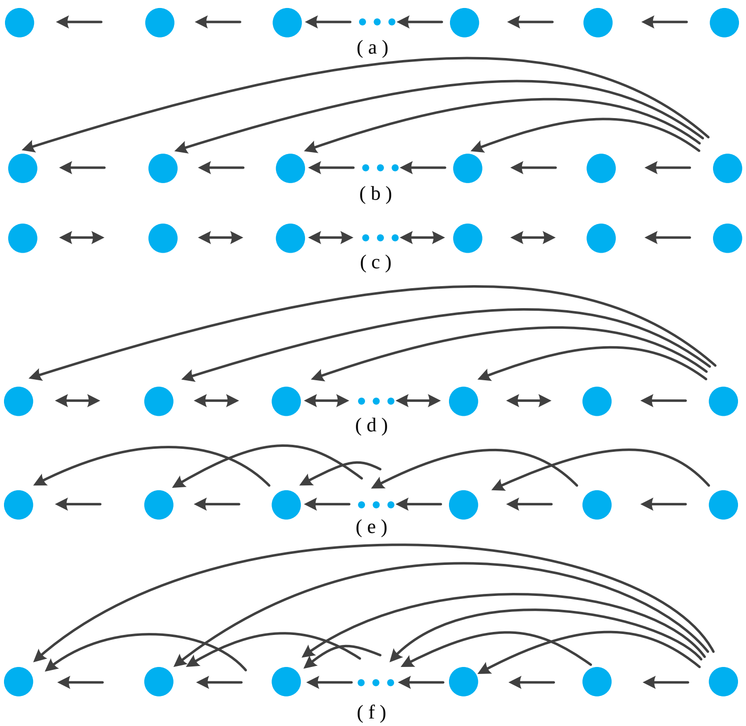
1. Introduction

View platoons from a networked control perspective

Typical Communication Topology

Connected Vehicle by V2V

Different Communication Topologies



New challenges

- New challenges naturally arise due to the **variety of topologies**, in particular when considering issues like nonlinear dynamic, input constraints etc.

How to design a distributed controller for a heterogeneous platoon considering nonlinear dynamics, input constraints and variety of communication topologies?

1. Introduction

Model Predictive Control

- ✓ MPC: control input is obtained by numerically optimizing a finite horizon optimal control problem
- ✓ Benefits: both nonlinearity and constraints can be explicitly handled
- ✓ It is typically used for a single-agent system.

Distributed MPC

Predicted trajectory
Assumed trajectory

Distributed MPC

1. W. B. Dunbar and R. M. Murray "Distributed receding horizon control for multi-vehicle formation stabilization", *Automatica*, vol. 42, no. 4, pp.549 -558, 2006.
2. T. Keviczky, F. Borrelli and G. J. Balas, "Decentralized receding horizon control for large scale dynamically decoupled systems", *Automatica*. vol. 42, no. 12, pp.2105 -2115, 2006.
3. H. Li, Y. Shi, “Distributed model predictive control of constrained nonlinear systems with communication delays”. *Systems & Control Letters*, vol. 62, no.10, pp. 819-826, 2013,
4. R R. Negenborn, J M Maestre. “Distributed model predictive control: An overview and roadmap of future research opportunities”. *Control Systems, IEEE*, vol. 34, no.4, pp. 87-97, 2014.

- ✓ The majority only focus on the **stabilization of the system with a common set point**
- ✓ Assuming all agents *a priori* know the **desired equilibrium information**.

For a vehicle platoon, such a common set point corresponds to the leader’s state.

Impractical

This work presents a DMPC algorithm for heterogeneous platoons with **unidirectional topologies** and ***a priori* unknown desired set point**.

Outline

2. Platoon model and controller design

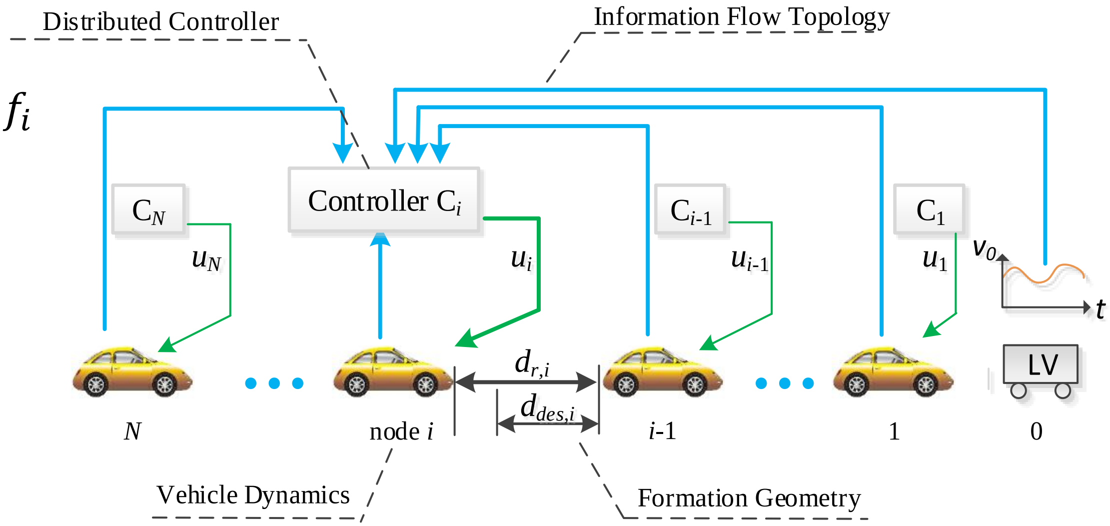
Vehicle Dynamics

$$\begin{cases} \dot{s}_i(t) = v_i(t) \\ \dfrac{\eta_{\mathrm{T},i}}{r_{\mathrm{w},i}} T_i(t) = m_i \dot{v}_i(t) + C_{\mathrm{A},i} v_i^2(t) + m_i g f_i \\ \tau_i \dot{T}_i(t) + T_i(t) = u_i(t) \end{cases}$$

Discrete Domain

$$\boldsymbol{x}_i(t+1) = \phi_i(\boldsymbol{x}_i) + \boldsymbol{\psi}_i \cdot u_i\,(t)$$

$s_i(t)$: postion; $v_i(t)$: speed;
$T_i(t)$: Torque ;



Control Objectives

a) to ensure the same speed with the leader
b) to maintain the desired spaces between adjacent vehicles

$$\begin{cases} \lim\limits_{t\to\infty} \|v_i(t) - v_0(t)\| = 0 \\ \lim\limits_{t\to\infty} \left\|s_{i-1}(t) - s_i(t) - d_{i-1,i}\right\| = 0 \end{cases}, i \in \mathcal{N}$$

$v_0(t)$: leader's speed

Constant spacing policy $d_{i-1,i} = d_0$

2. Platoon model and controller design

Communication Topology

Algebraic Graph Theory

- ✓ Viewed as a directed graph G, and use Pinning matrix, Adjacent matrix and Laplacian matrix to model the connections.
- ✓ The communication is assumed as to be perfect. There is no delay, data loss etc.

Definitions

- Pinning Matrix

To model the information flow from the leader to followers

$$\mathcal{P} = \begin{bmatrix} \wp_1 & & \\ & \ddots & \\ & & \wp_N \end{bmatrix}$$

$$\wp_i = 1 \,, if \;\; \{\alpha_0, \alpha_i\} \in E$$

- Adjacent Matrix

To model the information flow among followers

$$\mathcal{A}_N = [a_{ij}] \in \mathbb{R}^{N \times N}$$

$$\begin{cases} a_{ij} = 1, if \;\; \{\alpha_j, \alpha_i\} \in E \\ a_{ij} = 0, if \;\; \{\alpha_j, \alpha_i\} \notin E \end{cases}$$

- Laplacian Matrix

An induced matrix from adjacent matrix

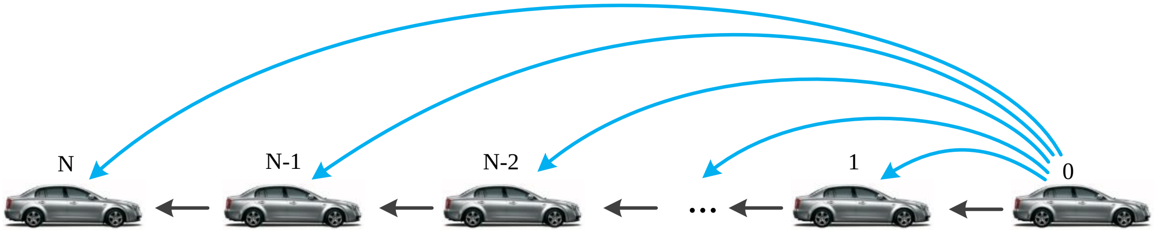
$$\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$$

$$l_{ij} = \begin{cases} -a_{ij} \quad , & i \neq j \\ \sum_{k=1}^{N} a_{ik} \, , & i = j \end{cases}.$$

2. Platoon model and controller design

Communication Topology

- Example: Predecessor-leader following Topology



$$\mathcal{P} = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} \qquad \mathcal{A}_N = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & \ddots & \ddots & \\ & & & 1 & 0 \end{bmatrix} \qquad \mathcal{L} = \begin{bmatrix} 0 & & & & \\ -1 & 1 & & & \\ & -1 & 1 & & \\ & & \ddots & \ddots & \\ & & & -1 & 1 \end{bmatrix}$$

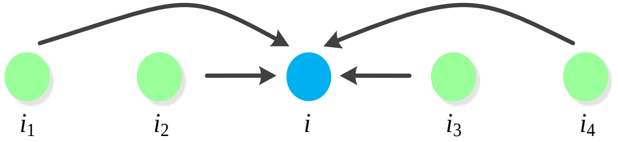
Some Definitions

1) **Spanning tree.** There exists a root node such that there is a directed path from this node to every other node.

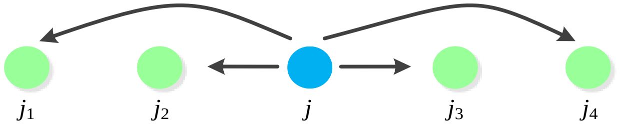
2) **Neighbor set.** $\mathbb{N}_i = \{j|a_{ij} = 1, j \in \mathcal{N}\}$

- The set $\mathbb{N}_i$ means that node i can receive the information of any $j \in \mathbb{N}_i$.

3) **Define a dual set** $\mathbb{O}_i = \{j|a_{ji} = 1, j \in \mathcal{N}\}$, which means that node i sends its information to any $j \in \mathbb{O}_i$.



$$\mathbb{N}_i = \{i_1, i_2, i_3, i_4\}$$



$$\mathbb{O}_j = \{j_1, j_2, j_3, j_4\}.$$

2. Platoon model and controller design

Design of DMPC algorithm

- ☐ Design **local open-loop optimal control problem** in each node;
- ☐ Every local problem can only use **the information of neighboring nodes** to compute its control input;
- ☐ Then they exchange information;

Some notations

The leader is assumed to run at a constant speed, *i.e.*,$s_0 = v_0 t$.

$$s_{\mathrm{des},i}(t) = s_0(t) - i \cdot d_0,$$
$$v_{\mathrm{des},i}(t) = v_0$$

Desired set point of state and input of node i

$$\begin{cases} x_{\mathrm{des},i}(t) = \left[s_{\mathrm{des},i}(t), v_{\mathrm{des},i}(t), T_{\mathrm{des},i}(t)\right]^T \\ u_{\mathrm{des},i}(t) = T_{\mathrm{des},i}(t), \end{cases}$$

Only for analysis

The nodes in $\mathbb{N}_i$ are numbered as $i_1, i_2, \cdots, i_m$

$$y_{-i}(t) = \left[y_{1_1}^T(t), y_{1_2}^T(t), \cdots, y_{1_m}^T(t)\right]^T,$$
$$u_{-i}(t) = \left[u_{i_1}(t), u_{i_2}(t), \cdots, u_{i_m}(t)\right]^T$$

Over the prediction horizon $[t, t+N_p]$, we define three types of trajectories:

$y_i^p(k|t)$: Predicted output trajectory,
$y_i^*(k|t)$: Optimal output trajectory,
$y_i^a(k|t)$: Assumed output trajectory

Three types of control inputs are also defined,

$u_i^p(k|t)$: Predicted control input,
$u_i^*(k|t)$: Optimal control input,
$u_i^a(k|t)$: Assumed control input

Standard process for DMPC

2. Platoon model and controller design

Local open-loop optimal control problem

Problem $\mathcal{F}_i$: For $i \in \{1,2,\ldots,N\}$ at time t

$$\min_{U_i} J_i\left(\boldsymbol{y}_i^p(:|t), u_i^p(:|t), \boldsymbol{y}_i^a(:|t), \boldsymbol{y}_{-i}^a(:|t)\right)$$

$$= \sum_{k=0}^{N_p-1} l_i\left(\boldsymbol{y}_i^p(k|t), u_i^p(k|t), \boldsymbol{y}_i^a(k|t), \boldsymbol{y}_{-i}^a(k|t)\right)$$

Cost function

s.t.

$$\dot{\boldsymbol{x}}_i^p(k+1|t) = \phi_i\left(\boldsymbol{x}_i^p(k|t)\right) + \boldsymbol{\psi}_i \cdot u_i^p(k|t),$$

$$\boldsymbol{y}_i^p(k|t) = \boldsymbol{\gamma}\boldsymbol{x}_i^p(k|t)$$

$$k = 0, \cdots, N_p - 1$$

Dynamic constraints in predictive horizon

$$\boldsymbol{x}_i^p(0|t) = \boldsymbol{x}_i(t)$$

$$u_i^p(k|t) \in \mathcal{U}$$

Input constraints

$$\boldsymbol{y}_i^p(N_p|t) = \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}\left(\boldsymbol{y}_j^a(N_p|t) - \tilde{\boldsymbol{d}}_{j,i}\right)$$

$$T_i^p(N_p|t) = h_i\left(v_i^p(N_p|t)\right)$$

Terminal Constraints→ stability

Move at constant speed at the end of predictive horizon

This is based on the local average of neighboring outputs. Thus, any node does not need to *a prior* know the desired set point,

2. Platoon model and controller design

Local open-loop optimal control problem

Construction of local cost function

Design Parameters

$$l_i\left(y_i^p(k|t), u_i^p(k|t), y_i^a(k|t), y_{-i}^a(k|t)\right)$$

$$= \left\| Q_i\left(y_i^p(k|t) - y_{\mathrm{des},i}(k|t)\right)\right\|_2$$

Tracking leader $p_i = 0,\ Q_i = 0$

$$+ \left\| R_i\left(u_i^p(k|t) - h_i\left(v_i^p(k|t)\right)\right)\right\|_2$$

Penalize the input $R_i \geq 0$

$$+ \left\| F_i\left(y_i^p(k|t) - y_i^a(k|t)\right)\right\|_2$$

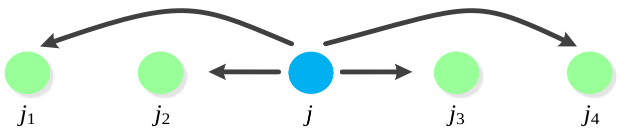
Maintain its assumed output $F_i \geq 0$

$$+ \sum_{j \in \mathbb{N}_i} \left\| G_i\left(y_i^p(k|t) - y_j^a(k|t) - \tilde{d}_{i,j}\right)\right\|_2$$

Maintain the assumed output of its neighbors $G_i \geq 0$

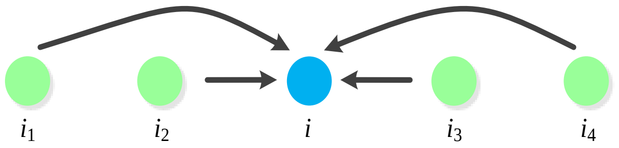
This output is sent to the nodes in set $\mathbb{O}_i$

Node i tries to maintain the output as close to the assumed trajectories of its neighbors (*i.e.*, $j \in \mathbb{N}_i$) as possible



$\mathbb{O}_j = \{j_1, j_2, j_3, j_4\}.$

Stability ⇔



$\mathbb{N}_i = \{i_1, i_2, i_3, i_4\}$

2. Platoon model and controller design

Local open-loop optimal control problem

Algorithm of distributed model predictive control

(I) Initialization At time $t = 0$, assume that all followers are moving at a constant speed

$$\begin{cases} u_i^a(k|0) = h_i\big(v_i(0)\big) \\ y_i^a(k|0) = y_i^p(k|0) \end{cases}, k = 0,1,\cdots,N_p - 1,$$

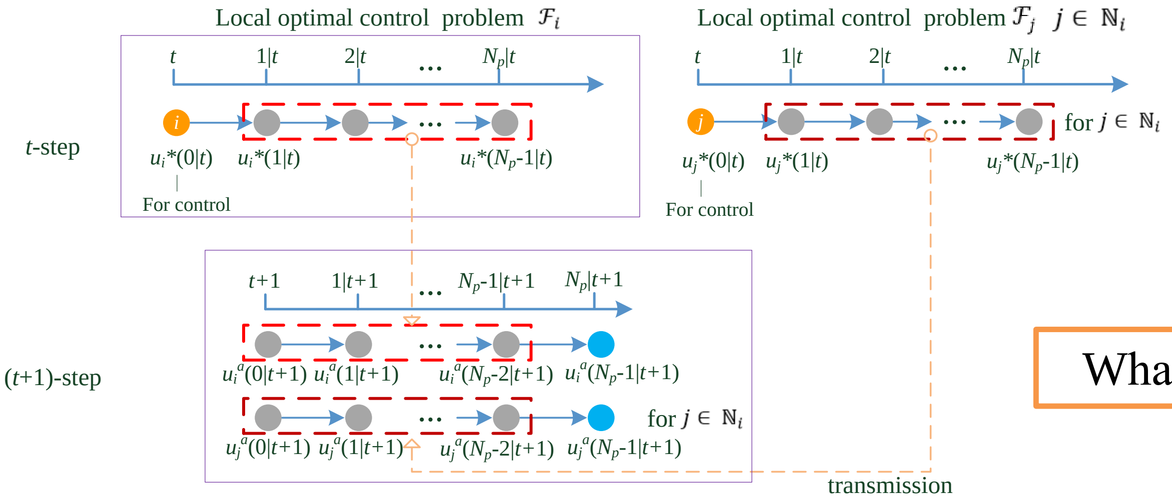
(II) Iteration of DMPC: At $t > 0$, for all node $i = 1,\cdots,N$

1. Optimize Problem $\mathcal{F}_i$, yielding optimal control sequence $u_i^*(k|t), k = 0,1,\cdots,N_p - 1$

2. Compute the assumed control input (*i.e.*, $u_i^a(k|t+1)$) for next step by disposing first term and adding one additional term

$$u_i^a(k|t+1) = \begin{cases} u_i^*(k+1|t), & k = 0,1,\cdots,N_p - 2 \\ h_i\left(v_i^*(N_p|t)\right), & k = N_p - 1 \end{cases}$$

3. Transmit $y_i^a(k|t+1)$ to the nodes in set $\mathbb{O}_i$, Receive $y_{-i}^a(k|t+1)$ from the nodes in set $\mathbb{N}_i$; Compute $y_{\mathrm{des},i}(k|t+1)$ if $\mathbb{P}_i \neq \emptyset$



4. Implement the control effort, *i.e.*, $u_i(t) = u_i^*(0|t)$

5. Increment t and go to step (1).

What's the requirement for stability?

Outline

3. Stability analysis of DMPC

Main strategy → standard

The main strategy is to construct a proper Lyapunov function for the platoon and prove its decreasing property → sum of local cost functions

Terminal Constraint ⇨ **Individual Local cost** ⇨ **Sum of local cost** ⇨ **Sufficient conditions**

Terminal Constraint → Recursive feasibility

Individual Local cost, Sum of local cost, Sufficient conditions: Monotonically decrease

Terminal constraint analysis

$$\boldsymbol{y}_i^p(N_p|t) = \frac{1}{|\mathbb{I}_i|}\sum_{j\in\mathbb{I}_i}\left(\boldsymbol{y}_j^a(N_p|t) - \tilde{\boldsymbol{d}}_{j,i}\right) \xrightarrow{\text{How ?}} \left\{\begin{array}{l} \lim\limits_{t\to\infty}\|v_i(t)-v_0(t)\| = 0 \\ \lim\limits_{t\to\infty}\left\|s_{i-1}(t)-s_i(t)-d_{i-1,i}\right\| = 0 \end{array}\right., i\in\mathcal{N}$$

$$\lim_{t\to\infty}\left|y_i^p(N_p|t) - y_{\text{des},i}(N_p|t)\right| = 0 \longrightarrow \boxed{\text{Consensus of terminal constraints}}$$

Theorem 1. If $\mathbb{G}$ contains a **spanning tree** rooting from the leader, the terminal state in the predictive horizon of problem $\mathcal{F}_i$ asymptotically converges to the desired state, *i.e.*,

$$\lim_{t\to\infty}\left|y_i^p(N_p|t) - y_{\text{des},i}(N_p|t)\right| = 0.$$

where $y_{\text{des},i}(N_p|t) = \left[s_0(N_p|t) - i\cdot d_0, v_0\right]^T$.

- ❑ A spanning tree is also a prerequisite to achieve a stable platoon.
- ❑ Intuitively, it means that every follower can obtain the leader information directly or indirectly.

3. Stability analysis of DMPC

Terminal Constraint ⇨ **Individual Local cost** ⇨ **Sum of local cost** ⇨ **Sufficient conditions**

Recursive feasibility

Monotonically decrease

◻ Terminal constraint analysis

Assumption 1 (Unidirectional topology): The graph $\mathbb{G}$ contains a spanning tree rooting at the leader, and the communications are unidirectional from preceding vehicles to downstream ones

Theorem 2. If $\mathbb{G}$ satisfies **Assumption 1**, the terminal state in the predictive horizon of problem $\mathcal{F}_i$ converges to the desired state at most N steps, *i.e.*,

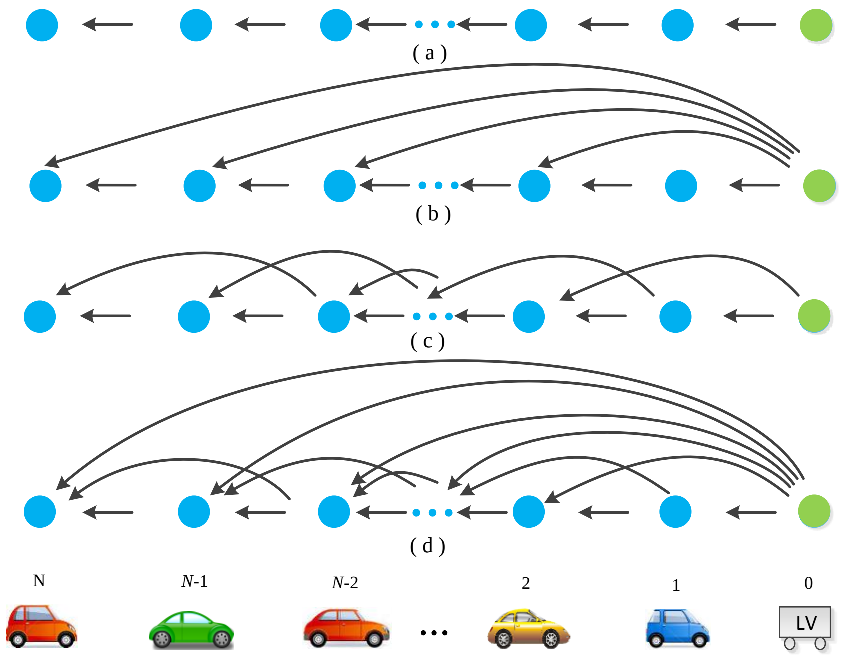
$$y_i^p(N_p|t) = y_{\mathrm{des},i}(N_p|t), \qquad t \geq N.$$

Lemma (Recursive feasibility). If we replace (13d) with $y_i^p(N_p|t) = y_{\mathrm{des},i}(N_p|t)$, then problem $\mathcal{F}_i$ has

$$\left(y_i^p(:|t), u_i^p(:|t)\right) = \left(y_i^a(:|t), u_i^a(:|t)\right)$$

as a feasible solution for any time $t > 0$

Suboptimal solution



3. Stability analysis of DMPC

Terminal Constraint ⇨ **Individual Local cost** ⇨ Sum of local cost ⇨ Sufficient conditions

Recursive feasibility

Monotonically decrease

Analysis of local cost function

Theorem 3. If $\mathbb{G}$ satisfies **Assumption 1**, each local cost function satisfies

$$J_i^*(t+1) - J_i^*(t) \le -l_i\left(y_i^*(0|t), u_i^*(0|t), y_i^a(0|t), y_{-i}^a(0|t)\right) + \varepsilon_i,$$
$$t > N$$

where

$$\varepsilon_i = \sum_{k=1}^{N_p-1}\left\{\sum_{j\in\mathbb{N}_i}\left\|G_i\left(y_j^*(k|t) - y_j^a(k|t)\right)\right\|_2 - \left\|F_i\left(y_i^*(k|t) - y_i^a(k|t)\right)\right\|_2\right\}$$

Sketch of Proof: at time $t+1, t \ge N$, a feasible control for $\mathcal{F}_i$ is $u_i^p(:|t+1) = u_i^a(:|t+1)$

$$J_i^*(t+1) \le J_i\left(y_i^a(:|t+1), u_i^a(:|t+1), y_i^a(:|t+1), y_{-i}^a(:|t+1)\right)$$

Give relationship between $J_i^*(t+1)$ and $J_i^*(t)$

It gives an upper bound on the decline of local cost function. If we have

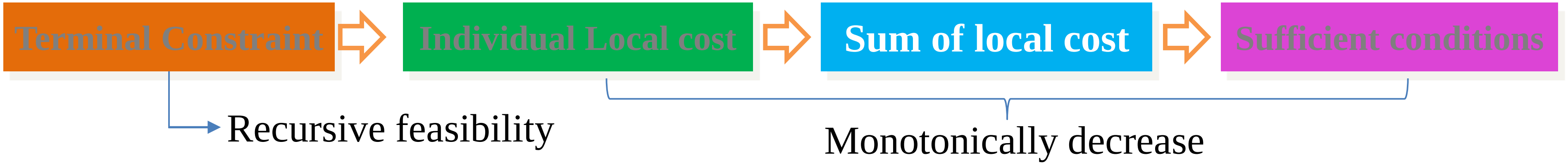
$$\varepsilon_i \le l_i\left(y_i^*(0|t), u_i^*(0|t), y_i^a(0|t), y_{-i}^a(0|t)\right)$$

Then the system is stable

there is no intuitive way to adjust control parameters.

sum of local cost functions

3. Stability analysis of DMPC


Terminal Constraint → Individual Local cost → **Sum of local cost** → Sufficient conditions

Recursive feasibility

Monotonically decrease

Sum of local cost functions

$$J_{\Sigma}^{*}(t)=\sum_{i=1}^{N} J_i^{*}\left(y_i^{*}(:|t),u_i^{*}(:|t),y_i^{a}(:|t),y_{-i}^{a}(:|t)\right)$$

Theorem 4. If 𝔾 satisfies **Assumption 1**, , J_Σ^* (t) satisfies

$$J_{\Sigma}^{*}(t+1)-J_{\Sigma}^{*}(t)\leq-\sum_{i=1}^{N} l_i\left(y_i^{*}(0|t),u_i^{*}(0|t),y_i^{a}(0|t),y_{-i}^{a}(0|t)\right)+\sum_{k=1}^{N_p-1}\varepsilon_{\Sigma}(k)\,,t>N$$

where

$$\varepsilon_{\Sigma}(k)=\sum_{i=1}^{N}\left[\sum_{j\in\mathbb{O}_i}\left\|G_j\left(y_i^{*}(k|t)-y_i^{a}(k|t)\right)\right\|_2-\left\|F_i\left(y_i^{*}(k|t)-y_i^{a}(k|t)\right)\right\|_2\right]$$

Sketch of Proof:

$$J_{\Sigma}^{*}(t+1)-J_{\Sigma}^{*}(t)\leq-\sum_{i=1}^{N} l_i\left(y_i^{*}(0|t),u_i^{*}(0|t),y_i^{a}(0|t),y_{-i}^{a}(0|t)\right)+\sum_{i=1}^{N}\varepsilon_i\,.$$

To change $\mathbb{N}_i$ to $\mathbb{O}_i$ by considering all followers in the platoon

3. Stability analysis of DMPC

Terminal Constraint → Individual Local cost → Sum of local cost → Sufficient conditions

Recursive feasibility

Monotonically decrease

☐ Sufficient conditions

Theorem 5. If G satisfies Assumption 1, a platoon under proposed DMPC is asymptotically stable if satisfying

Maintain its assumed output

$$F_i \geq \sum_{j\in\mathbb{O}_i} G_j, \quad i \in \mathcal{N}$$

Maintain the assumed output of its neighbors

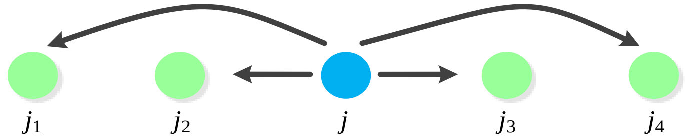
Sketch of Proof:

$$z^T\left(\sum_{j\in\mathbb{O}_i} G_j - F_i\right) z \leq 0, \forall\, z \in \mathbb{R}^2$$

strictly monotonically decreasing

$$J_\Sigma^*(t+1) - J_\Sigma^*(t) \leq -\sum_{i=1}^{N} l_i\left(y_i^*(0|t), u_i^*(0|t), y_i^a(0|t), y_{-i}^a(0|t)\right)$$

To ensure stability implies that all nodes in $\mathbb{O}_i$ should not rely heavily on the information of node *i* unless node *i* shows good-enough consistence with its own assumed trajectory

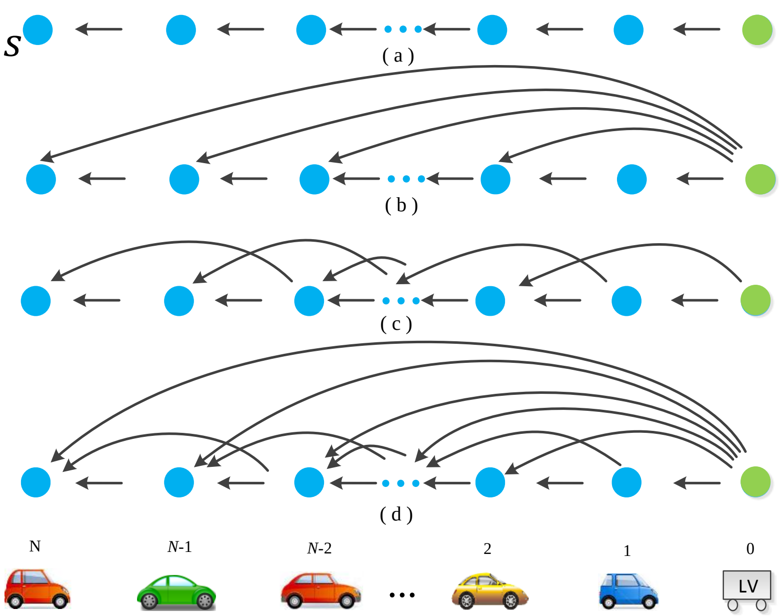


$$\mathbb{O}_j = \{j_1, j_2, j_3, j_4\}.$$

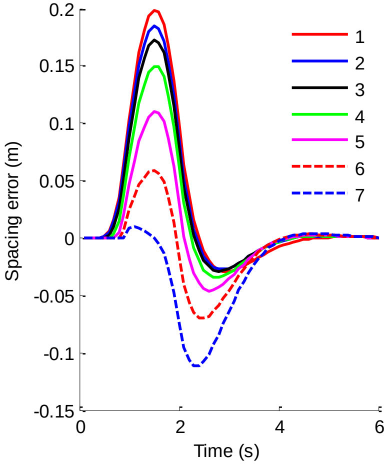
Outline

4. Simulation results

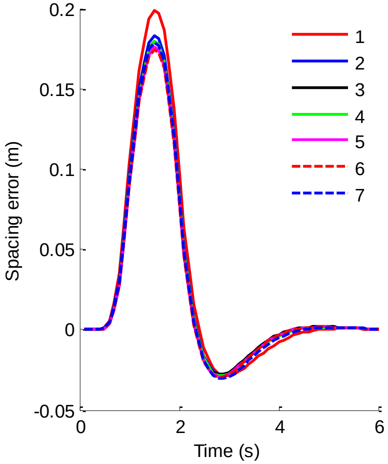
The desired trajectory
$$v_0 = \begin{cases} 20\ m/s & t \le 1\ s \\ 20 + 2t\ \ m/s & 1s <\ t \le 2\ s \\ 22\ m/s & t > 2s \end{cases}$$



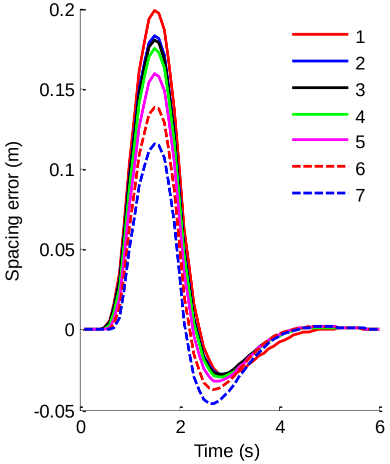
Weights	PF	PLF	TPF	TPLF
F_i	$F_i = 10I_2$, $i \in \mathcal{N}$	$F_i = 10I_2$, $i \in \mathcal{N}$	$F_i = 10I_2$, $i \in \mathcal{N}$	$F_i = 10I_2$, $i \in \mathcal{N}$
G_i	$G_1 = 0$, $G_i = 5I_2$, $i \in \mathcal{N}\backslash\{1\}$	$G_1 = 0$, $G_i = 5I_2$, $i \in \mathcal{N}\backslash\{1\}$	$G_1 = 0$, $G_i = 5I_2$, $i \in \mathcal{N}\backslash\{1\}$	$G_1 = 0$, $G_i = 5I_2$, $i \in \mathcal{N}\backslash\{1\}$
Q_i	$Q_1 = 10I_2$, $Q_i = 0$, $i \in \mathcal{N}\backslash\{1\}$	$Q_i = 10I_2$, $i \in \mathcal{N}$	$Q_1 = 10I_2$, $Q_2 = 10I_2$, $Q_i = 0$, $i \in \mathcal{N}\backslash\{1,2\}$	$Q_i = 10I_2$, $i \in \mathcal{N}$
R_i	$R_i = I_2$, $i \in \mathcal{N}$	$R_i = I_2$, $i \in \mathcal{N}$	$R_i = I_2$, $i \in \mathcal{N}$	$R_i = I_2$, $i \in \mathcal{N}$



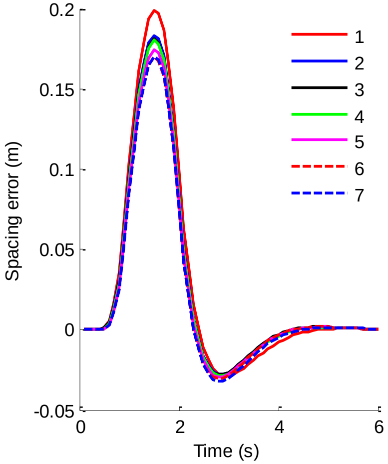
(a) PF



(b) PLF



(c) TPF



(d) TPLF

Outline

5. Conclusions

Key points

- This work proposes a novel DMPC algorithm for vehicle platoons with nonlinear dynamics and unidirectional topologies
- A sufficient condition is derived to guarantee asymptotic stability.
- This approach does not require all vehicles *a priori* know the desired set point

Consensus of terminal state ⇨ **Asymptotic stability for the whole system**

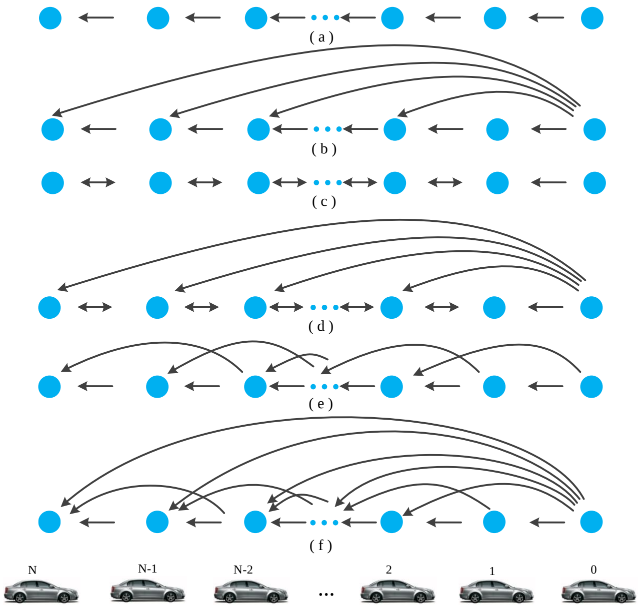
- ✓ Spanning tree → infinite time
- ✓ Unidirectional topology → finite time

- ✓ An explicit sufficient condition on the weights of the cost functions

Future work

1. Topology aspect → spanning tree ?
2. Communication issue → Time delay ?
3. Computational issue → feasibility & efficiency ?
4. Dynamics → uncertainty & robustness DMPC ?

This work is submitted to IEEE Transactions on Control Systems Technology



Distributed Model Predictive Control for Heterogeneous Vehicle Platoons under Unidirectional Topologies

Thank you! Q&A